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# Method for Calculating Force Coefficients of Bodies of Revolution

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**Localized projectile–fluid interaction models are widely used for calculating drag and lift coefficients in a high-velocity flow in free-molecular, intermediate, and continuum flow regimes. A fast procedure for evaluating drag and lift coefficients of bodies of revolution in the range of the angles of attack from 0 to 90 deg, which can be employed in these calculations, is suggested. The developed exact method reduces computation of integrals from local force coefficients over the exposed part of the projectile's surface at each value of angle of attack to the evaluation one-dimensional quadratures. Such simplification is achieved by analyzing all possible types of the exposed surface structure and derivation of the analytical formulas via integration over one of the coordinates. The developed procedure comprises a set of formulas and tables, which can be readily implemented in a computer code. Method is applied in an intermediate flow region with a localized interaction model often used in hypersonic aerodynamics and the obtained results are compared with experimental data.**

## Nomenclature

$\boldsymbol{a}$ = vector of global parameters characterizing projectile–fluid interaction
$a_i^p, a_i^\tau, a_i^D, a_i^L$ = parameters determining a specific localized interaction model
$C$ = integral aerodynamic force coefficient
$C_D$ = drag coefficient
$C_L$ = lift coefficient
$D$ = diameter of projectile base, m
$L$ = projectile length, m
$\boldsymbol{n}^\circ$ = local inner normal vector at a body's surface
$R$ = radius of a projectile base, m; see Fig. 1
$Re_0$ = Reynolds number based on $D$
$R^p, R^\tau, R^D, R^L$ = parameters determining a specific localized interaction model
$S_0$ = characteristic area, area of a projectile base, m$^2$
$S_*$ = exposed area, m$^2$
$T_W$ = surface temperature of a projectile, K
$T_0$ = stagnation temperature, K
$\boldsymbol{v}_\infty^\circ$ = freestream velocity, m/s
$\boldsymbol{v}_{\infty\perp}^\circ$ = vector perpendicular to $\boldsymbol{v}_\infty$; see Fig. 1
$x, y, z$ = coordinates; see Fig. 1
$\alpha$ = angle of attack
$\gamma$ = specific heats ratio
$\rho, \theta$ = coordinates; see Fig. 1
$\boldsymbol{\tau}^\circ$ = tangent vector at a surface point in the plane determined by vectors $\boldsymbol{n}^\circ$ and $\boldsymbol{v}_\infty^\circ$
$\phi(\rho)$ = function determining a longitudinal contour of a projectile
$\Omega_p, \Omega_\tau$ = functions determining a specific model of a projectile–flow interaction
$\omega$ = angle between vectors $\boldsymbol{n}^\circ$ and $\boldsymbol{v}_\infty^\circ$

*Superscripts*

$\circ$ = unit vector
$\dot{\phi}$ = derivative with respect to $\rho$

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## Introduction

DURING the last two decades various localized interaction models were developed and used at different physical conditions for calculating the momentum and energy exchange between a moving projectile and the surrounding gaseous medium.[1,2] In these models it is assumed that the total effect of projectile–fluid interaction can be represented as a sum of independent local interactions of small surface elements of the projectile with a medium. These local interactions depend on the local geometric and kinematic characteristics of the surface and on global parameters, which are the same for all projectile surfaces. In the most widely used version of this approach, applicable for translational motion of a projectile, the only local parameter is the angle between the main stream velocity vector and the local unit normal vector to the surface. Specific models of this type (e.g., see Ref. 1) were used in hypersonic aerodynamics in free molecular, intermediate, and continuum flow regimes.

Because of the wide use of the local interaction models it is desirable to develop a calculation procedure that is more efficient than direct integration of the local interactions over the exposed surface. This motivated the differential equations method,[1–5] which reduces the problem to solving the recurrent system of ordinary differential equations. However, determining the particular solutions and constants in the general solutions of these differential equations is very cumbersome. A method suggested in this study appears to be more convenient in application since it allows the derivation of explicit formulas for aerodynamic coefficients through one-dimensional quadratures.

## Formulation of the Problem

In general, localized interaction models can be represented mathematically as follows[1]:

$$\boldsymbol{c}_F = \Omega_p(\boldsymbol{a}, \omega)\boldsymbol{n}^\circ + \Omega_\tau(\boldsymbol{a}, \omega)\boldsymbol{\tau}^\circ \tag{1}$$

where $\boldsymbol{c}_F$ is surface force coefficient vector per unit surface area.

The specific model is determined by functions $\Omega_p$ and $\Omega_\tau$, and the parameters $\boldsymbol{a}$ have physical meaning (e.g., Mach number, Reynolds number, etc.). The integral force coefficient is determined by integrating $\boldsymbol{c}_F$ over the exposed surface area of the body,

$$\boldsymbol{C} = \frac{1}{S_0}\iint_{S_*} \boldsymbol{c}_F \, \mathrm{d}s \tag{2}$$

where $S_*$ is determined by the condition

$$\cos\omega \geq 0 \tag{3}$$

The effect of the flow on the shaded area of the surface can be also taken into account.

The models that are determined by the equations

$$\Omega_p = \sum_{i=0}^{R^p} a_i^p \cos^i \omega, \qquad \Omega_\tau = \sin\omega \sum_{i=0}^{R^\tau} a_i^\tau \cos^i \omega \tag{4}$$

and which were suggested in Refs. 3 and 4 are of particular interest. These models can be used as exact or as approximate models in the high-velocity continuum regime,[6] intermediate and free molecular regimes of rarefied gas flows.[4,6–8] The typical example of such models is a model suggested in Ref. 8, which has been successfully used in hypersonic aerodynamics for calculating aerodynamic characteristics of nonslender bodies in intermediate flow regime:

$$\Omega_p = a_2^p \cos^2 \omega + a_1^p \cos\omega, \qquad \Omega_\tau = a_1^\tau \sin\omega \cos\omega \tag{5}$$

where

$$a_1^p = \left[\frac{\pi(\gamma-1)}{\gamma}\right]^{\frac{1}{2}} \exp[-Re_0(0.125 + 0.078t_w)], \quad a_2^p = 2 \tag{6a}$$

$$a_1^\tau = 5.22[Re_1 + 6.88\exp(Re_2)]^{-\frac{1}{2}} \tag{6b}$$

$$Re_2 = (0.0072 - 0.000016Re_1)Re_1 \tag{7a}$$

$$Re_1 = Re_0(0.75t_w + 0.25)^{-0.67}, \qquad t_w = T_w/T_0 \tag{7b}$$

The coefficients of viscosity in the formula for the Reynolds number $Re_0$ is calculated at temperature $T_0$. At $Re_0 \to 0$, the model determined by Eqs. (5–7) recovers the formulas of a free molecular flow with diffuse reflection, and at $Re_0 \to \infty$, it is equivalent to Newton's model for a continuum flow regime.

The coordinate systems and designations used are presented in Fig. 1. Hereafter we present all of the coordinates in dimensionless form with $R$ used as a characteristic length. The equation of the surface of an axisymmetric body can be written as follows:

$$x = \phi(\rho), \qquad \phi(0) = 0 \tag{8}$$

where

$$\dot\phi > 0, \qquad \ddot\phi > 0, \qquad 0 < \rho \le 1 \tag{9}$$

Formulas for aerodynamic force coefficients for the general model described by Eq. (1) read

$$C_D = \boldsymbol{C}\cdot \boldsymbol{v}_\infty^0 = \frac{1}{\pi}\iint_\sigma \rho\Omega_D(\boldsymbol{a},\omega)t_0^{\frac{1}{2}}\,\mathrm{d}\rho\,\mathrm{d}\theta \tag{10}$$

$$C_L = \boldsymbol{C}\cdot \boldsymbol{v}_{\infty\perp}^0 = \frac{1}{\pi}\iint_\sigma \rho\Omega_L(\boldsymbol{a},\omega)t_2\,\mathrm{d}\rho\,\mathrm{d}\theta \tag{11}$$

where $\sigma$ is a projection of the exposed part of the projectile surface on a $yz$ plane and

$$\Omega_D = \cos\omega\Omega_p + \sin\omega\Omega_\tau, \qquad \Omega_L = \Omega_p - \cot\omega\Omega_\tau \tag{12}$$

$$\cos\omega = t = t_1/t_0^{\frac{1}{2}}, \qquad t_1 = \cos\alpha - \dot\phi\sin\alpha\cos\theta \tag{13a}$$

$$t_0 = \dot\phi^2 + 1, \qquad t_2 = -\sin\alpha - \dot\phi\cos\alpha\cos\theta \tag{13b}$$

Consider now the structure of the exposed surface. Using Eqs. (13a) and (13b), the condition of exposure given by Eq. (3) reduces to $t_1 \ge 0$. When $\alpha = 0$ the lateral surface of a projectile is completely exposed. For $0 < \alpha < \pi/2$ it is convenient to represent the condition for exposure in the following form:

$$\dot\phi\cos\theta \le \cot\alpha \tag{14}$$

In the following we use inequalities implied by Eq. (9):

$$\dot\phi(0) < \dot\phi(\rho) < \dot\phi(1), \qquad 0 < \rho < 1 \tag{15}$$

Let the angle of attack be such that

$$\cot\alpha \ge \dot\phi(1) \tag{16}$$

Since Eq. (15) implies that $\dot\phi\cos\theta \le \dot\phi(1)$, Eq. (14) is satisfied at any point of the lateral surface, i.e., it is completely exposed. In the following this range of the angles of attack is called range 1 (see Table 1).

Let the angle of attack be such that

$$\dot\phi(0) < \cot\alpha < \dot\phi(1) \tag{17}$$

When $\rho$ increases from 0 to $\rho_*$, where

$$\rho_* = \rho_*(\alpha) = \dot\phi^{-1}(\cot\alpha) \tag{18}$$

then $\dot\phi(\rho)$ increases from $\dot\phi(0)$ to $\cot\alpha$ and Eq. (14) is satisfied for arbitrary $0 \le \theta \le 2\pi$. Therefore, the part of the lateral surface with $\rho \le \rho_*$ is completely exposed. When $\rho > \rho_*$, then $\cot\alpha/\dot\phi(\rho) < 1$ and Eq. (14) determines the boundaries of the exposed part of the surface $\theta = \theta_*, \theta = 2\pi - \theta_*$, where

$$\theta_* = \theta_*(\alpha,\rho) = \cos^{-1}[\cot\alpha/\dot\phi(\rho)] \tag{19}$$

In the following the latter range of the angles of attack is called range 2 (see Table 1). In this case the exposed part of the surface consists of two domains.

Finally, let the angle of attack be such that

$$\cot\alpha \le \dot\phi(0) \tag{20}$$

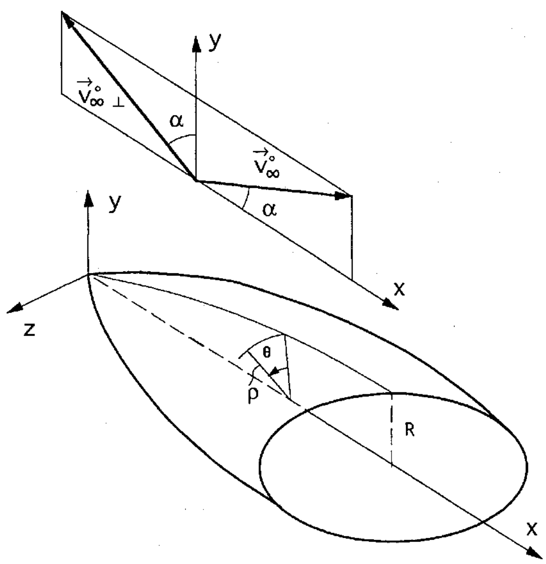


**Fig. 1 Coordinates and notations.**

**Table 1 Structure of exposed surface area**

| Range number | Range of variation of angle of attack | Exposed area | $W_k^T$ |
|---|---|---|---|
| 1 | $0 \le \alpha \le \alpha_1$ | $0 \le \rho \le 1, \quad 0 \le \theta \le 2\pi$ | $H_k^T(0, 1, 0)$ |
| 2 | $\alpha_1 \le \alpha < \alpha_2$ | $0\, 0 \le \rho \le \rho_*, \quad 0 \le \theta \le 2\pi$ | $H_k^T(0, \rho_*, 0)$ |
| | | $\rho_* < \rho \le 1, \quad \theta_* \le \theta \le 2\pi - \theta_*$ | $H_k^T(\rho_*, 1, \theta_*)$ |
| 3 | $\alpha_2 \le \alpha < \pi/2$ | $0 \le \rho \le 1, \quad \theta_* \le \theta \le 2\pi - \theta_*$ | $H_k^T(0, 1, \theta_*)$ |

Then for arbitrary $\rho$ the situation is the same as within the range 2 when $\rho > \rho_*$. In the following, this range of the angles of attack is called range 3 (see Table 1).

The results of the analysis are presented in the first three columns of Table 1, where

$$\alpha_1 = \cot^{-1}[\dot{\phi}(0)], \qquad \alpha_2 = \cot^{-1}[\dot{\phi}(0)] \tag{21}$$

Obviously, for a blunt body, where $\dot{\phi}(0) = 0$, only ranges 1 and 2 occur.

Hereafter all of the derivations are performed for the model determined by Eq. (4). Substituting formulas for $\Omega_p$ and $\Omega_\tau$ in Eq. (4) into Eqs. (10) and (11) and using Eqs. (12), (13a), and (13b), we arrive at the following expressions for the force coefficients:

$$C_D = \frac{2}{\pi}\sum_{k=0}^{R^D} a_k^D W_k^D \tag{22}$$

$$C_L = \frac{2}{\pi}\sum_{k=0}^{R^L} a_k^L W_k^L \tag{23}$$

Formulas for functions $W_k^T$ ($T = D, L$) in various ranges of the angles of attack are shown in the fourth column in Table 1 and

$$H_k^D(\rho_-, \rho_+, \beta) = \int_{\rho_-}^{\rho_+} \rho t_0^{(1-k)/2} J_k^D \mathrm{d}\rho \tag{24}$$

$$J_k^D(\beta) = \int_\beta^\pi t_1^k \,\mathrm{d}\theta \tag{25}$$

$$H_k^L(\rho_-, \rho_+, \beta) = \int_{\rho_-}^{\rho_+} \rho t_0^{-k/2} J_k^L \mathrm{d}\rho \tag{26}$$

$$J_k^L(\beta) = \int_\beta^\pi t_1^k t_2 \,\mathrm{d}\theta \tag{27}$$

$$a_i^L = a_i^p - a_{i-1}^\tau, \qquad a_j^D = a_{j-1}^p + a_j^\tau - a_{j-2}^\tau \tag{28}$$

where coefficients $a_i^p$ with subscripts $i < 0$ or $i > R^p$ and coefficients $a_j^\tau$ with subscripts $j < 0$ or $j > R^\tau$ are assumed to be equal to zero. Parameters $R^D$ and $R^L$ are chosen so that starting from $i = R^D + 1$ and $j = R^L + 1$ and all $a_i^D$ and $a_j^L$ are equal to zero.

Thus, the problem of calculating force coefficients is reduced to derivation formulas for calculating $W_k^D$ and $W_k^L$ ($k = 0, 1, \ldots$).

## Evaluation of Drag Coefficients

Inspection of Table 1 shows that to determine $W_k^D$, formulas for calculating $H_k^D$ for $\beta = 0$ and for $\beta = \theta_*(\alpha, \rho)$ must be derived. After calculating the integrals in Eq. (25), we obtain

$$J_k^D(0) = \pi \sum_{i=0}^{k_0} B_{ki}^{(0)} \frac{(\sin\alpha)^{2i}(\cos\alpha)^{k-2i}}{\dot{\phi}^{2i}} \tag{29}$$

$$J_k^D(\theta_*) = \left(1 - \frac{\theta_*}{\pi}\right) J_k^D(0) + \sqrt{\dot{\phi}^2 - \cot^2\alpha} \sum_{i=0}^{k_1} B_{ki}^{(1)} \frac{(\sin\alpha)^{2i+1}(\cos\alpha)^{k-2i-1}}{\dot{\phi}^{2i}} \tag{30}$$

where

$$k_0 = [k/2], \qquad k_1 = [(k-1)/2] \tag{31}$$

$$B_{ki}^{(0)} = \frac{k!}{(k-2i)!\,(i!)^2\, 2^{2i}}, \qquad C_{nm} = \frac{n!}{m!\,(n-m)!} \tag{32}$$

$$B_{ki}^{(1)} = \sum_{j=i+1}^{k_1+1} \left[\frac{C_{k,2j-1}}{2j-1} \times \sigma_{ij} - \frac{(2j-1)C_{k,2j}}{2j(2j-2i-1)} \times \frac{1}{\sigma_{ij}}\right] \tag{33}$$

Table 2 Calculation of functions $W_k^D$ and $W_{k-1}^L$

| Range of angle of attack | $W_k^D$ | $W_{k-1}^L$ |
|---|---|---|
| 1 | $W_k^D(0, 0)$ | $W_{k-1}^L(0, 0)$ |
| 2 | $W_k^D(1, \rho_*)$ | $W_{k-1}^L(1, \rho_*)$ |
| 3 | $W_k^D(1, 0)$ | $W_{k-1}^L(1, 0)$ |

$$\sigma_{ij} = \begin{cases} 1 & \text{if } i = 0 \\ \dfrac{2^i(j-1)(j-2)\cdots(j-i)}{(2j-3)(2j-5)\cdots(2j-2i-1)} & \text{if } i > 0 \end{cases} \tag{34}$$

and [•] denotes the integer part of the number in brackets. When $k = 0$, the second sum in Eq. (30) is considered to equal zero.

Substituting $J_k^D$ from Eqs. (29) and (30) into Eq. (24) and using the identity

$$\frac{u^i}{(u+1)^m} = \sum_{\mu=m-i}^{m} (-1)^{m+i-\mu} C_{i,m-\mu}(u+1)^{-\mu} \tag{35}$$

where $i$ and $m$ are positive integers yields formulas for calculating the function $W_k^D$, which are shown in Table 2 for all of the ranges of $\alpha$ ($\alpha > 0$), where

$$W_k^D(\xi, \rho_-) = \sum_{\mu=0}^{k_0} g_{k\mu}^{(0)}[\pi K_\mu(\Delta, 0) - \xi L_\mu(\Delta, \rho_-)] + \xi \sum_{\mu=\mu_0}^{k_0} g_{k\mu}^{(1)} M_\mu(\Delta, \rho_-) \tag{36}$$

where $\xi = 0, 1$.

The second sum in Eq. (36) must be set equal to zero if $k = 0$; $\mu_0 = 1$, $\Delta = \frac{1}{2}$ for even $k$ and $\mu_0 = 0$, $\Delta = 0$ for odd $k$ and

$$K_\mu(\Delta, \rho_-) = \int_{\rho_-}^1 \psi \,\mathrm{d}\rho, \qquad \psi = \rho t_0^{\Delta-\mu} \tag{37}$$

$$L_\mu(\Delta, \rho_-) = \int_{\rho_-}^1 \theta_* \psi \,\mathrm{d}\rho \tag{38}$$

$$M_\mu(\Delta, \rho_-) = \int_{\rho_-}^1 \sqrt{\dot{\phi}^2 - \cot^2\alpha}\,\psi \,\mathrm{d}\rho \tag{39}$$

$$g_{k\mu}^{(\delta)} = \sum_{i=k_0-\mu}^{k_\delta} \gamma_{k\mu i}^{(\delta)} (\sin\alpha)^{2i+\delta}(\cos\alpha)^{k-2i-\delta} \tag{40}$$

$$\gamma_{k\mu i}^{(\delta)} = (-1)^{k_0+i-\mu} C_{i,k_0-\mu} B_{ki}^{(\delta)}, \qquad \delta = 0, 1 \tag{41}$$

Thus the determination of the drag coefficient reduces to the calculation of integrals in Eqs. (37–39). For convenience the values of the coefficients $\gamma_{k\mu i}^{(\delta)}$ are given in Table 3 in the form of fractions, where $\gamma_{k\mu i}^{(0)}$ is in the numerator and $\gamma_{k\mu i}^{(1)}$ is in the denominator.

## Calculation of Lift Coefficient

The integrals in Eq. (27) can be simplified by using the equation[1,2,5]

$$W_{k-1}^L = \frac{1}{k}\frac{\partial}{\partial\alpha} W_k^D, \qquad k = 1, 2, \ldots \tag{42}$$

Thus, the problem of calculating lift coefficient is reduced to calculating the derivatives of function $W_k^D$. Therefore, integrals must be calculated with their upper limits depending on parameter $\alpha$. However, the derivative of the integral is equal to the integral of the derivative since the integrand vanishes after substitution of the

**Table 3 Values of $\gamma_{k\mu i}^{(\delta)}$**

| $i/k$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| | | | | $\mu = 0$ | | | |
| 0 | $\frac{1.00000}{—}$ | $\frac{1.00000}{1.00000}$ | | | | | |
| 1 | | | $\frac{0.50000}{—}$ | $\frac{1.50000}{0.66667}$ | | | |
| 2 | | | | | $\frac{0.37500}{—}$ | $\frac{1.87500}{4.80000}$ | |
| 3 | | | | | | | $\frac{0.31250}{—}$ |
| | | | | $\mu = 1$ | | | |
| 0 | | | $\frac{1.00000}{1.50000}$ | $\frac{1.00000}{1.83333}$ | | | |
| 1 | | | $\frac{1.00000}{—}$ | $\frac{-1.50000}{-0.66667}$ | $\frac{3.00000}{2.29167}$ | $\frac{5.00000}{7.19167}$ | |
| 2 | | | | | $\frac{-0.75000}{—}$ | $\frac{-3.75000}{-9.60000}$ | $\frac{5.62500}{28.76528}$ |
| 3 | | | | | | | $\frac{-0.93750}{—}$ |
| | | | | $\mu = 2$ | | | |
| 0 | | | | | $\frac{1.00000}{2.08333}$ | $\frac{1.00000}{2.28333}$ | |
| 1 | | | | | $\frac{-3.00000}{-2.29167}$ | $\frac{-5.00000}{-7.19167}$ | $\frac{7.50000}{22.08519}$ |
| 2 | | | | | $\frac{-0.37500}{—}$ | $\frac{1.87500}{4.80000}$ | $\frac{-11.25000}{-57.53055}$ |
| 3 | | | | | | | $\frac{0.93750}{—}$ |
| | | | | $\mu = 3$ | | | |
| 0 | | | | | | | $\frac{1.00000}{2.45000}$ |
| 1 | | | | | | | $\frac{-7.50000}{-22.08519}$ |
| 2 | | | | | | | $\frac{5.62500}{28.76528}$ |
| 3 | | | | | | | $\frac{-0.31250}{—}$ |

upper limit. After some algebra we obtain the formulas that are shown in Table 2 for all ranges of $\alpha > 0$, where

$$W_{k-1}^{D}(\xi, \rho_-) = \frac{1}{k}\sum_{\mu=0}^{k_0}\left\{\tilde{g}_{k\mu}^{(0)}[\pi K_\mu(\Delta, 0) - \xi L_\mu(\Delta, \rho_-)] - \frac{\xi}{\sin^2\alpha} g_{k\mu}^{(0)} N_\mu(\Delta, \rho_-)\right\} + \frac{\xi}{k}\sum_{\mu=\mu_0}^{k_0}\left\{\tilde{g}_{k\mu}^{(1)} M_\mu(\Delta, \rho_-) + \frac{\cos\alpha}{\sin^3\alpha} g_{k\mu}^{(1)} N_\mu(\Delta, \rho_-)\right\} \tag{43}$$

$$N_\mu(\Delta, \rho_-) = \int_{\rho_-}^{1} (\dot{\phi}^2 - \cot^2\alpha)^{-\frac{1}{2}} \psi \, \mathrm{d}\rho \tag{44}$$

$$\tilde{g}_{k\mu}^{(\delta)} = \frac{\partial}{\partial\alpha} g_{k\mu}^{(\delta)} = \sum_{i=k-\mu}^{k_0} \gamma_{k\mu i}^{(\delta)} (k\cos^2\alpha + \delta + 2i - k) \frac{(\sin\alpha)^{2i+\delta-1}}{(\cos\alpha)^{2i-k+\delta+1}} \tag{45}$$

Thus determination of the lift coefficient $C_L$ requires calculation of the singular integrals in Eq. (44), which converge.

## Application Procedure

At small values of the angles of attack (range 1), integrals $K_\mu$ depend on the shape of a projectile but not on $\alpha$. Therefore, the derived formulas are convenient for analysis of analytical expressions for aerodynamic coefficients $C_D$ and $C_L$ as functions of the angle of attack. In the whole range of angles of attack the described procedure requires numerical implementation. Parameters of the model $a_i^n$, $a_j^\tau$, $R^n$, and $R^\tau$ are assumed to be known. For the reader's convenience an algorithm of such a procedure is presented.

Step 1) Calculate coefficients $a_k^D$ and $a_k^L$ using Eq. (28).
Step 2) Calculate integrals $K_\mu$ from Eq. (37).
Step 3) Calculate $\gamma_{k\mu i}^{(\delta)}$ from Eq. (41) or Table 3.
The following steps must be performed for each value of $\alpha$:

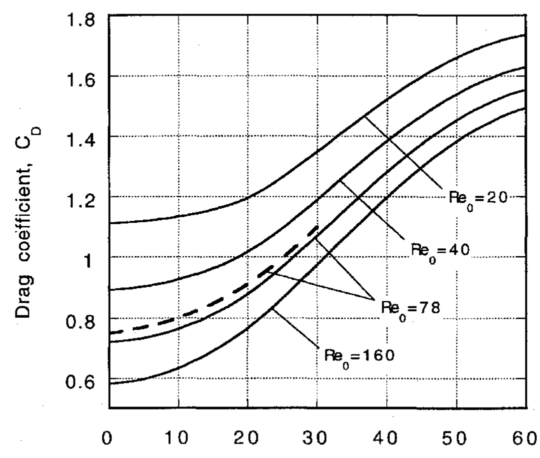


**Fig. 2 Drag coefficient vs angle of attack for power law body, $D/L$ = 0.8, $m$ = 4/3: solid lines, present theory and broken lines, experiments.**

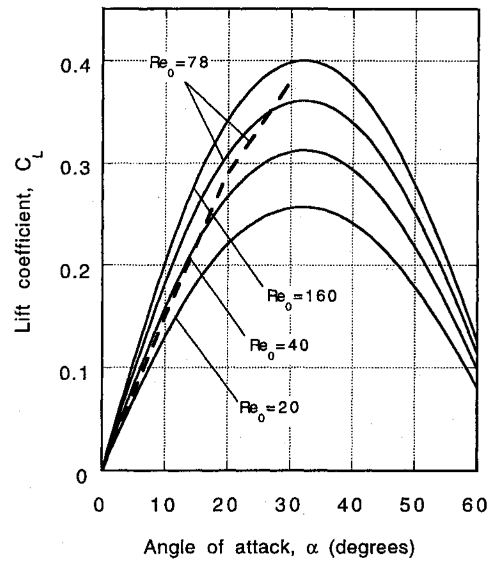


**Fig. 3 Lift coefficient vs angle of attack for power law body, $D/L = 0.8$, $m = 4/3$: solid lines, present theory and broken lines, experiments.**

Step 4) Calculate $q_{k\mu}^{(\delta)}$ from Eq. (40).
Step 5) Calculate integrals $L_\mu$, $M_\mu$, $N_\mu$ using Eqs. (38) and (39) ($\rho_- = \rho_*$ if $\alpha$ belongs to range 2 and $\rho_- = 0$ if $\alpha$ belongs to range 3).
Step 6) Calculate $W_k^D$ and $W_k^L$ using Table 2.
Step 7) Calculate $C_D$ and $C_L$ using Eqs. (22) and (23).

This procedure was used in a numerical code and the obtained results were compared with those calculated using other methods. In Figs. 2 and 3 we present results for a hypersonic rarefied gas flow in the intermediate regime over a projectile with a power law shape given by the following equation:

$$x = (2L/D)\rho^{\frac{4}{3}} \tag{46}$$

Calculations were performed using the model determined by Eqs. (5–7) with $\gamma = 1.4$ and $t_w = 1$. The obtained results agree fairly well with the experimental results[9] (experimental error about 6%).

## Conclusions

An explicit handy-to-use method is derived to calculate force coefficients for bodies of revolution as functions of the angle of attack. The derived formulas can be used for analytical analysis and numerical computations with localized interaction models used in high-speed aerodynamics.

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